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Numerical Study of a Fluid Dynamic Traffic Flow Model

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Abstract-Fluid dynamic traffic flow model considered as macroscopic model is a mathematical model that formulates the relationships among traffic flow characteristics like density, flow, mean speed of a traffic stream etc. We consider a fluid dynamic traffic flow model first developed by Lighthill and Whitham (1955) and Richard (1956) shortly called LWR traffic flow model. In this paper, we study two finite difference schemes such as first order explicit upwind difference scheme- EUDS (forward time backward space) and second order Lax-Wendroff difference scheme-LWDS (forward time centered space) for solving first order PDE of LWR macroscopic traffic flow model appended with initial and boundary conditions. The traffic density $\rho(t,x)$ is computed by solving LWR macroscopic conservative form of traffic flow model using both schemes. Stability conditions of the schemes are determined and it is numerically shown that LWDS is superior to EUDS in terms of time step selection. The conditions of stability are also numerically verified. Some numerical simulation results are presented for various parameters.

Keywords: LWR Macroscopic Traffic Flow Model, Finite Difference Method and Numerical Simulation.

1 Introduction

Traffic flow can be defined as the study of how the vehicles move between origin and destination, and how the individual drivers interact with others. Since the driver behavior cannot be predicted with absolute certainty, mathematical models have been built which study the consistent behavior between the traffic streams via relationships such as flow q, density ρ and the mean velocity v ([1], [2]). The continuum traffic flow model was the first order model developed by Lighthill, Whitham (1955) and Richards (1956) ([4], [9]) based on the assumption of mass density conservation, that is, the number of vehicles between any two points if there are no entrances or exits is conserved. The LWR model is a first-order model in the sense it is formulated as a scalar hyperbolic conservation law, and is often solved by finite difference methods ([5], [6], [7], [8]). The non-linear first order partial differential is appended by initial and boundary value leads to an initial boundary value problem (IBVP). It is too complex to be solved by analytical methods. We can be solved the non-linear PDE by the method of characteristics as a Cauchy problem. However, with the rapid development of numerical methods and computer technology the system can be solved numerically. Numerical solution of the non-linear first order partial differential equation of traffic flow is obtained by using explicit upwind difference scheme (EUDS) and Lax-Wendroff difference scheme (LWDS) with initial and boundary value. The traffic density $\rho(t,x)$ is computed using both schemes. The conditions of stability are also numerically verified.

2 Governing Equation of LWR Traffic Flow Model

The general mathematical equation of traffic flow model with the initial condition reads as initial value problem (IVP) ([3], [4], and [5]) is

$$\frac{\partial\rho}{\partial t}+\frac{\partial}{\partial x}\left(v_{\max}\left(\rho-\frac{\rho^2}{\rho_{\max}}\right)\right)=0 \quad (1)$$

$$\text{with } \rho(t_0,x)=\rho_0(x)$$

3 Exact Solution of the Non-linear PDE of LWR Traffic Flow Model

The non-linear PDE of IVP (1) can be solved [6] by the method of characteristics. The exact solution of the IVP (1) is given by [5]

$$\rho(t,x)=\rho_0\left(x-v_{\max}\left(1-\frac{2\rho}{\rho_{\max}}\right)t\right) \quad (2)$$

which is non-linear implicit form and therefore very complicated to evaluate at each $\rho(t,x)$. However, in reality it is very difficult to approximate the initial density $\rho_0(x)$ of the Cauchy problem (1) as a function of t from given initial data. Therefore, there is a demand of some efficient numerical methods for solving the IVP (1).

4 Finite Difference Method of LWR Traffic Flow Model

We consider our specific non-linear partial differential equation of LWR traffic flow model as an initial boundary value problem (IBVP):

$$\frac{\partial \rho}{\partial t}+\frac{\partial}{\partial x}\big(q(\rho)\big)=0, t_o \le t \le T,\ a \le x \le b$$

$$\text{with i.c. } \rho(t_o, x)=\rho_o(x);\ a \le x \le b \qquad (3)$$

$$\text{and b.c. } \rho(t,a)=\rho_a(t);\ t_o \le t \le T,$$

$$\text{where } q(\rho)=v_{\max}\left(\rho-\frac{\rho^2}{\rho_{\max}}\right).$$

Finite difference methods are the efficient approach to numerical solutions of partial differential equations. A finite difference method proceeds by replacing the derivatives in the differential equation by the finite difference approximation. This gives a large algebraic system of equation to be developing a computer programming code.

4.1 Explicit Upwind Difference Scheme by FTBS Techniques

In this section, we study a finite difference scheme for the non-linear LWR traffic flow model. In order to develop the scheme, we discretize the space and time. We discretize the time derivative $\frac{\partial \rho}{\partial t}$ and space derivative $\frac{\partial q}{\partial x}$ in the IBVP (3) at any discrete point (t_n, x_i) for $i=1,......,M$ and $n=0,.......,N-1$. We assume the uniform grid spacing $t^{n+1}=t^n+k$ and $x_{i+1}=x_i+h$.

The discretization of $\frac{\partial \rho}{\partial t}$ is obtained by first order forward difference in time $\frac{\partial \rho}{\partial t}=\frac{\rho_i^{n+1}-\rho_i^n}{\Delta t}$ (4)

Next use the backward space difference formula $\frac{\partial q}{\partial x}=\frac{q_i^n-q_{i-1}^n}{\Delta x}$ (5)

Substituting equation (4), (5) into equation (3) and writing ρ_i^n for $\rho(t^n, x_i)$, the discrete version of the non-linear PDE of LWR traffic flow model formulates the first order explicit upwind difference scheme of the form

$$\frac{\rho_i^{n+1}-\rho_i^n}{\Delta t}+\frac{q(\rho_i^n)-q(\rho_{i-1}^n)}{\Delta x}=0$$

$$\Rightarrow \rho_i^{n+1}=\rho_i^n-\frac{\Delta t}{\Delta x}\big(q(\rho_i^n)-q(\rho_{i-1}^n)\big) \qquad (6)$$

$$\text{where } q(\rho_i^n)=v_{\max}\left(\rho_i^n-\frac{(\rho_i^n)^2}{\rho_{\max}}\right)$$

Also equation (6) can be written as

$$\rho_i^{n+1}=\rho_i^n-q'(\rho_i^n)\frac{\Delta t}{\Delta x}\big(\rho_i^n-\rho_{i-1}^n\big)$$

$$\Rightarrow \rho_i^{n+1}=(1-\lambda)\rho_i^n+\lambda\rho_{i-1}^n;\ \text{where } \lambda := q'(\rho_i^n)\frac{\Delta t}{\Delta x} \qquad (7)$$

4.2 Lax-Wendroff Difference Scheme by FTCS Techniques

In order to develop the 2nd order Lax-Wendroff method, named after P. Lax and B. Wendroff, can be derived in terms of the discrietization of $\frac{\partial \rho}{\partial t}$ is obtained by first order forward difference in time $\frac{\partial \rho}{\partial t}=\frac{\rho_i^{n+1}-\rho_i^n}{\Delta t}$.

Next use the discretization of $\frac{\partial q}{\partial x}$ is obtained by second order centered space difference formula. From the Taylor's series expansion we can write

$$\rho(x,\ t+k)=\rho(x,t)+k\frac{\partial \rho}{\partial t}+\frac{k^2}{2!}\frac{\partial^2 \rho}{\partial t^2}+ \qquad (8)$$

$$\text{i.e. } \frac{\partial \rho(t^n, x_i)}{\partial t}\approx\frac{\rho_i^{n+1}-\rho_i^n}{\Delta t}$$

$$\text{and } \frac{\partial}{\partial x}\big(q(\rho)(t^n, x_i)\big)\approx\frac{q(\rho_{i+1}^n)-q(\rho_{i-1}^n)}{2\Delta x} \qquad (9)$$

Now in equation (8), where the time derivatives can be replaced space derivatives using

$$\rho_t+\big(q(\rho)\big)_x=0 \qquad (10)$$

This has been done by so called Cauchy-Kawalewski technique which implies

$$\frac{\partial \rho}{\partial t} = -\frac{\partial q(\rho)}{\partial x}.$$

$$\therefore \frac{\partial^2 \rho}{\partial t^2} = \frac{\partial}{\partial t}\left(-\frac{\partial q(\rho)}{\partial x}\right) = \frac{\partial}{\partial x}\left(-\frac{\partial q(\rho)}{\partial t}\right)$$

$$= \frac{\partial}{\partial x}\left(-q'(\rho)\frac{\partial \rho}{\partial t}\right) = \frac{\partial}{\partial x}\left(-q'(\rho).-\frac{\partial q(\rho)}{\partial x}\right)$$

$$= \frac{\partial}{\partial x}\left(q'(\rho)\frac{\partial q(\rho)}{\partial x}\right);\text{ where } q'(\rho) = \frac{\partial q(\rho)}{\partial \rho}.$$

Substitute the preceding expression of time derivatives (10) into the Taylor's series of $\rho(x, t+k)$ in equation (8) to obtain

$$\rho(x,\, t+k) = \rho(x,t) - k\frac{\partial q(\rho)}{\partial x} + \frac{k^2}{2!}\frac{\partial}{\partial x}\left(q'(\rho)\frac{\partial q(\rho)}{\partial x}\right) + o(\Delta t^3) \quad (11)$$

Using equation (9), we get

$$\frac{\partial}{\partial x}\left(q'(\rho)\frac{\partial q(\rho)}{\partial x}\right)\left(t^n, x_i\right) =$$

$$\frac{q'\left(\rho^n_{i+\frac{1}{2}}\right)\left(q\left(\rho^n_{i+1}\right) - q\left(\rho^n_i\right)\right) - q'\left(\rho^n_{i-\frac{1}{2}}\right)\left(q\left(\rho^n_i\right) - q\left(\rho^n_{i-1}\right)\right)}{(\Delta x)^2} + o(\Delta x^2)$$

From equation (8) we get,

$$\frac{\rho(x, t+k) - \rho(x,t)}{k} = -\frac{\partial q(\rho)}{\partial x} + \frac{k}{2!}\frac{\partial}{\partial x}\left(q'(\rho)\frac{\partial q(\rho)}{\partial x}\right)$$

$$\Rightarrow \frac{\partial \rho}{\partial t} = -\frac{\partial q(\rho)}{\partial x} + \frac{k}{2!}\frac{\partial}{\partial x}\left(q'(\rho)\frac{\partial q(\rho)}{\partial x}\right)$$

$$\Rightarrow \frac{\partial \rho}{\partial t}\left(t^n, x_i\right) = -\frac{\partial q(\rho)}{\partial x}\left(t^n, x_i\right) + \frac{k}{2!}\frac{\partial}{\partial x}\left(q'(\rho)\frac{\partial q(\rho)}{\partial x}\right)\left(t^n, x_i\right)$$

$$\Rightarrow \frac{\rho^{n+1}_i - \rho^n_i}{\Delta t} = -\frac{1}{2\Delta x}\left(q(\rho^n_{i+1}) - q(\rho^n_{i-1})\right) + \frac{\Delta t}{2!}\frac{q'\left(\rho^n_{i+\frac{1}{2}}\right)\left(q\left(\rho^n_{i+1}\right) - q\left(\rho^n_i\right)\right) - q'\left(\rho^n_{i-\frac{1}{2}}\right)\left(q\left(\rho^n_i\right) - q\left(\rho^n_{i-1}\right)\right)}{(\Delta x)^2}$$

$$\therefore \rho^{n+1}_i = \rho^n_i - \frac{1}{2}\left(\frac{\Delta t}{\Delta x}\right)\left(q(\rho^n_{i+1}) - q(\rho^n_{i-1})\right) + \frac{1}{2}\left(\frac{\Delta t}{\Delta x}\right)^2\begin{pmatrix} q'\left(\rho^n_{i+\frac{1}{2}}\right)\left(q\left(\rho^n_{i+1}\right) - q\left(\rho^n_i\right)\right) \\ -q'\left(\rho^n_{i-\frac{1}{2}}\right)\left(q\left(\rho^n_i\right) - q\left(\rho^n_{i-1}\right)\right)\end{pmatrix}$$

(12)

where

$$q'\left(\rho^n_{i\pm\frac{1}{2}}\right) = v_{\max}\left(1 - \frac{2.\frac{1}{2}\left(\rho^n_{i\pm1} + \rho^n_i\right)}{\rho_{\max}}\right)$$

$$= v_{\max}\left(1 - \frac{1}{\rho_{\max}}\left(\rho^n_{i\pm1} + \rho^n_i\right)\right)$$

and

$$q\left(\rho^n_{i+1}\right) = v_{\max}\left(\rho^n_{i+1} - \frac{\left(\rho^n_{i+1}\right)^2}{\rho_{\max}}\right),\ q\left(\rho^n_i\right) = v_{\max}\left(\rho^n_i - \frac{\left(\rho^n_i\right)^2}{\rho_{\max}}\right),$$

$$q\left(\rho^n_{i-1}\right) = v_{\max}\left(\rho^n_{i-1} - \frac{\left(\rho^n_{i-1}\right)^2}{\rho_{\max}}\right).$$

4.3 Stability Conditions

The implementation of EUDS and LWDS is not straight forward. Since vehicles are moving in only one direction, so the characteristic speed $\frac{dq}{dt}$ must be positive. Stability condition of EUDS is determined in [5] that is the well-posed-ness and stability condition of the explicit finite difference scheme (7) is guaranteed by the simultaneous conditions

$$\rho_{\max} = k \max_i \rho_{\circ}(x_i),\ k \geq 2 \text{ and } \frac{v_{\max}\,\Delta t}{\Delta x} \leq 1.$$

Also the stability condition for second order LWDS is

$$\rho^{n+1}_i = \rho^n_i - \frac{1}{2}\left(\frac{\Delta t}{\Delta x}\right)\left(q(\rho^n_{i+1}) - q(\rho^n_{i-1})\right) + \frac{1}{2}\left(\frac{\Delta t}{\Delta x}\right)^2 q'\left(\rho^n_{i+\frac{1}{2}}\right)\left(q\left(\rho^n_{i+1}\right) - q\left(\rho^n_i\right)\right)$$

$$-\frac{1}{2}\left(\frac{\Delta t}{\Delta x}\right)^2 q'\left(\rho^n_{i-\frac{1}{2}}\right)\left(q\left(\rho^n_i\right) - q\left(\rho^n_{i-1}\right)\right)$$

$$= \rho^n_i - \frac{1}{2}\left(\frac{\Delta t}{\Delta x}\right)q'\left(\rho^n_i\right)\left(\rho^n_{i+1} - \rho^n_{i-1}\right) + \frac{1}{2}\left(\frac{\Delta t}{\Delta x}\right)^2 q'\left(\rho^n_{i+\frac{1}{2}}\right)q'\left(\rho^n_i\right)\left(\rho^n_{i+1} - \rho^n_i\right)$$

$$-\frac{1}{2}\left(\frac{\Delta t}{\Delta x}\right)^2 q'\left(\rho^n_{i-\frac{1}{2}}\right)q'\left(\rho^n_i\right)\left(\rho^n_i - \rho^n_{i-1}\right)$$

$$= \left(r + r_2\right)\rho^n_{i-1} + \left(1 - r_1 - r_2\right)\rho^n_i + \left(r_1 - r\right)\rho^n_{i+1};$$

$$r = \frac{1}{2}\left(\frac{\Delta t}{\Delta x}\right)q'\left(\rho^n_i\right),\ r_1 = \frac{1}{2}\left(\frac{\Delta t}{\Delta x}\right)^2 q'\left(\rho^n_{i+\frac{1}{2}}\right)q'\left(\rho^n_i\right) \text{ and } r_2 = \frac{1}{2}\left(\frac{\Delta t}{\Delta x}\right)^2 q'\left(\rho^n_{i-\frac{1}{2}}\right)q'\left(\rho^n_i\right)$$

This equation implies that if

$$0 \le r + r_2 \le 1 \quad (13)$$

$$0 \le 1 - r_1 - r_2 \le 1 \quad (14)$$

$$0 \le r_1 - r \le 1 \quad (15)$$

then the new solution is a convex combination of the two previous solutions. That is the solution at new time-step $(n+1)$ at a spatial node is an average of the solutions at the previous time-step at the spatial nodes $i-1,\ i$ and $i+1$. This means that the extreme value of the new solution is the average of the extreme values of the previous two solutions at the three consecutive nodes. Therefore, the new solution continuously depends on the initial value $\rho_i^o,\ i = 1,2,3,\ldots\ldots,M$.

Since $r_2 = \frac{1}{2}\left(\frac{\Delta t}{\Delta x}\right)^2 q'\left(\rho^n_{i-\frac{1}{2}}\right) q'\left(\rho_i^n\right)$

$$= \frac{1}{2}\left(\frac{\Delta t}{\Delta x}\right)^2 (v_{\max})^2 \left(1 - \frac{\left(\rho_{i-1}^n + \rho_i^n\right)}{\rho_{\max}}\right)\left(1 - \frac{2\rho_i^n}{\rho_{\max}}\right)$$

$$= \frac{1}{2}\left(\frac{\Delta t}{\Delta x}\right)^2 (v_{\max})^2 \left(1 - \frac{2\max(\rho_i^o)}{\rho_{\max}}\right)$$

$$\left(1 - \frac{2\max(\rho_i^o)}{\rho_{\max}}\right)$$

$$= \frac{1}{2}\left(\frac{\Delta t}{\Delta x} v_{\max}\right)^2 \left(1 - \frac{2\max(\rho_i^o)}{\rho_{\max}}\right)^2$$

Similarly,

$$r_1 = \frac{1}{2}\left(\frac{\Delta t}{\Delta x} v_{\max}\right)^2 \left(1 - \frac{2\max(\rho_i^o)}{\rho_{\max}}\right)^2.\ i.e.\ r_1 = r_2.$$

Equation (14) implies,

$$0 \le 1 - r_2 \le 1 \Rightarrow -1 \le -2r_2 \le 0 \Rightarrow 1 \ge 2r_2 \ge 0$$

$$\Rightarrow 0 \le 2r_2 \le 1 \Rightarrow 0 \le r_2 \le \frac{1}{2}$$

$$\Rightarrow 0 \le \frac{1}{2}\left(\frac{\Delta t}{\Delta x} v_{\max}\right)^2 \left(1 - \frac{2\max(\rho_i^o)}{\rho_{\max}}\right)^2 \le \frac{1}{2}$$

$$\Rightarrow 0 \le \left(\frac{\Delta t}{\Delta x} v_{\max}\right)\left(1 - \frac{2\max(\rho_i^o)}{\rho_{\max}}\right) \le 1$$

$$\Rightarrow 0 \le \left(\frac{\Delta t}{\Delta x} v_{\max}\right) \le 1 \Big/ \left(1 - \frac{2\max(\rho_i^o)}{\rho_{\max}}\right).$$

and equation (13) implies,

$$0 \le r + r_2 \le 1 \Rightarrow 0 \le r + \frac{1}{2} \le 1 \Rightarrow -\frac{1}{2} \le r \le \frac{1}{2}$$

$$\Rightarrow -\frac{1}{2} \le \frac{1}{2}\left(\frac{\Delta t}{\Delta x}\right) q'\left(\rho_i^n\right) \le \frac{1}{2} \Rightarrow -1 \le \left(\frac{\Delta t}{\Delta x}\right) q'\left(\rho_i^n\right) \le 1$$

$$\Rightarrow -\Delta x \le \Delta t\, q'\left(\rho_i^n\right) \le \Delta x \Rightarrow -\Delta x \le \Delta t\, v_{\max}\left(1 - \frac{2\rho_i^n}{\rho_{\max}}\right) \le \Delta x$$

$$\Rightarrow -\Delta x \le \Delta t\ v_{\max} \max\left(\rho_i^o\right) \le \Delta x.$$

4.4 Computational Results and Discussion

We implement two numerical finite difference schemes that are first order Explicit Upwind Difference Scheme (EUDS) and second order Lax-Wendroff Difference Scheme (LWDS) by computer programming and perform numerical simulation as described below.

4.4.1 Comparative Profile of Traffic Density in Different Time Step

We present numerical simulation results based on first order i.e. explicit upwind difference scheme (EUDS) and second order Lax-Wendroff difference scheme (LWDS). **Figure-1** shows density profiles of exact solution in different time step when $v_{\max} = 60\,\text{km/hour}$. **Figure-2** shows comparison of density among exact solution, EUDS and LWDS in 600th, 1200th and 1800th time step. From the figure we see that density profiles of LWDS are close nearer to exact solution and EUDS is close to LWDS but not nearer to exact solution. In **figure-3** discretization parameters $\Delta t = 0.1$ and $\Delta x = 0.2$ solid line represents the exact solution, the dot line represents the EUDS and the red line represents LWDS of density profile in last time step and we see that density profile in right boundary red line has enough jigjag. In discretization parameters

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$\Delta t=0.6$ and $\Delta x=0.4$ **figure-4** last time step jigjag is no more than figure-3. In **figure-5** discretization parameter $\Delta t=0.05$ and $\Delta x=0.04$ we see that right boundary has very few jigjag. Finally, when discretization parameter $\Delta t=0.01$ and $\Delta x=0.04$ **figure-6** shows the density profile has no jigjag. From above discretization parameters satisfying the stability conditions of EUDS and LWDS.

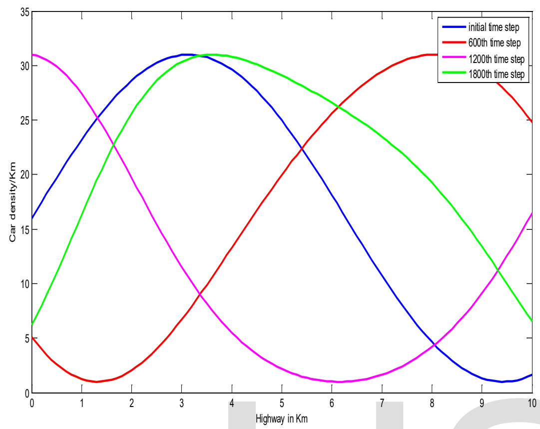

Figure-1: Density profile of exact solution in different time step when $v_{\max} = 60\,\text{km/hour}$

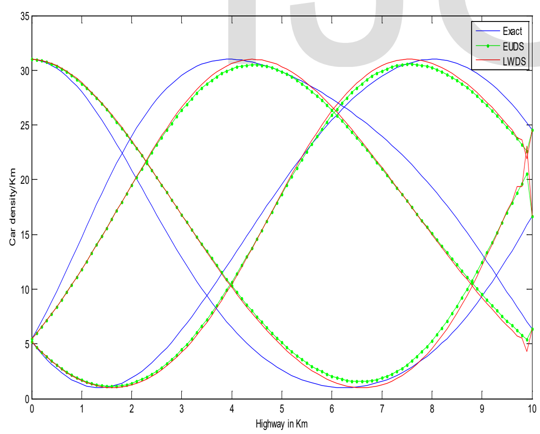

Figure-2: Comparison density profile of exact solution, EUDS and LWDS in 600^{th}, 1200^{th}, 1800^{th} time step

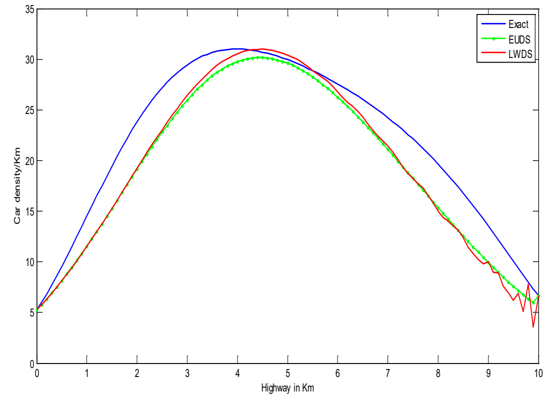

Figure 3: Comparison density profile of exact solution, EUDS and LWDS in last time step when $\Delta t=0.1$ and $\Delta x=0.2$

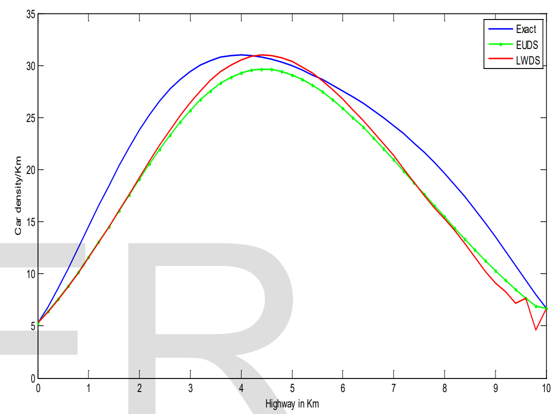

Figure 4: Comparison density profile of exact solution, EUDS and LWDS in last time step when $\Delta t=0.6$ and $\Delta x=0.4$

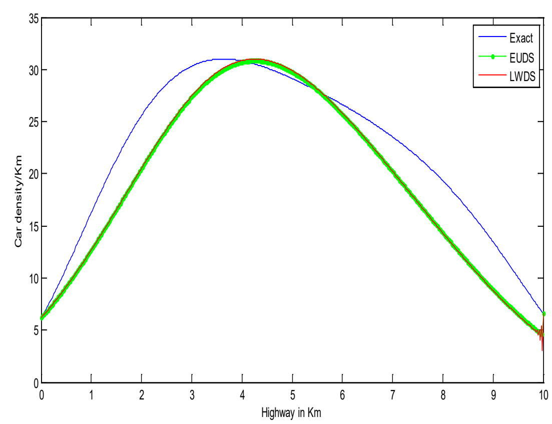

Figure-5: Comparison density profile of exact solution, EUDS and LWDS in last time step when $\Delta t=0.05$ and $\Delta x=0.04$

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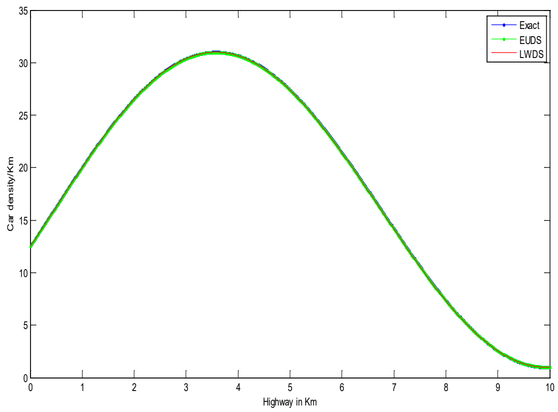

Figure-6: Comparison density profile exact solution, EUDS and LWDS in last time step when $\Delta t = 0.01$ and $\Delta x = 0.04$

4.4.2 Comparison Error Estimation and Convergence of Numerical Schemes

In order to perform error estimation for density (ρ), we consider exact solution (2) with initial condition i.e. linear function $\rho_0(x) = \frac{1}{2}x$, we have

$$\rho(t,x) = \rho_0(x_0) = \frac{1}{2}\left(x - v_{\max}\left(1 - \frac{2\rho}{\rho_{\max}}\right)t\right)$$

$$\Rightarrow \rho(t,x) = \frac{(x - v_{\max} t)/2}{(1 - v_{\max} t)/\rho_{\max}}.$$

We prescribe the corresponding boundary value for EUDS by the equation

$$\rho_a(t) = \rho(t,x_a) = \frac{(x_a - v_{\max} t)/2}{(1 - v_{\max} t)/\rho_{\max}}$$

and also the corresponding two sided boundary values for LWDS by the equations

$$\rho_a(t) = \rho(t,x_a) = \frac{(x_a - v_{\max} t)/2}{(1 - v_{\max} t)/\rho_{\max}}$$

and $\rho_b(t) = \rho(t,x_b) = \dfrac{(x_b - v_{\max} t)/2}{(1 - v_{\max} t)/\rho_{\max}}$

We compute the relative error in L_1-norm defined by $\|e\|_1 = \dfrac{\|\rho_e - \rho_n\|_1}{\|\rho_e\|_1}$ for all time ρ_e is the exact solution and ρ_n is the numerical solution computed by finite difference scheme.

Figure-7 comparison of relative errors for density (ρ) between explicit upwind difference scheme and Lax-Wendroff difference scheme which shows that Lax-Wendroff difference scheme provides more accurate results than explicit upwind difference scheme. **Figure-8** present that the density (ρ) error is decreasing with respect to the smaller descritization parameters Δt and Δx which shows the convergence of explicit upwind difference scheme and Lax-Wendroff difference scheme. We observe that as we increase number of grid points the error is decreasing and also shows the rate of convergence of the numerical solutions.

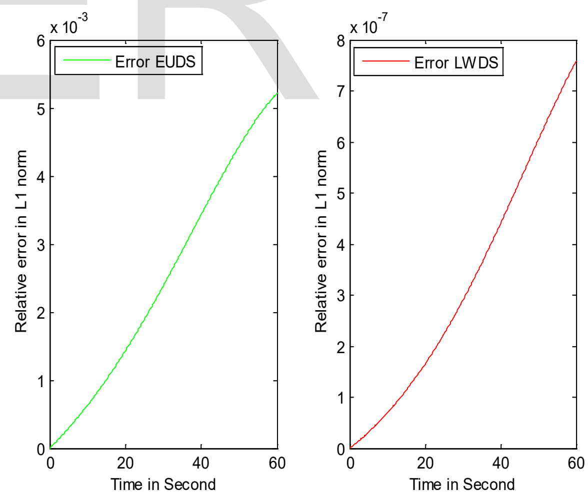

Figure-7: Comparison of relative errors between EUDS and Lax-Wendroff difference scheme

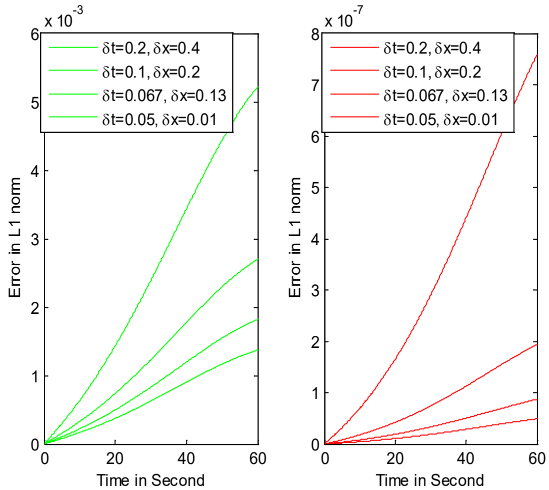

Figure-8: Convergence of errors between EUDS and Lax-Wendroff difference scheme

5 Conclusion

We have demonstrated exact solution and numerical solution by using EUDS and LWDS. We establish stability conditions of EUDS and LWDS. From the above figure it is seen that LWDS density profile is very close to exact solution. Also we observe that the relative error of LWDS is much less than that of EUDS and the rate of convergence of LWDS is much higher than that of EUDS which is due to the fact that LWDS is a second order model while the order of EUDS is one.

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